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Kinematic Analysis of Rotary Deep-Depth Turning Parameters

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Abstract. This article offers a parameterization procedure for deep-depth turning without simplifying assumptions. In this paper the authors will show comparative results of researching parameters for rotary turning according to the developed methodology and according to the methodology of finish-machining conditions. A theoretically found kinematic coefficient of rotary cutting is presented in the paper

1. Introduction

Several investigators have made analytical researching of the parameters of rotary turning. However, not all the authors solved this task in relation to finish machining with the help of simplifying assumptions. If not to doubt the integrity of the obtained results when analyzing "shallow-depth" ($t \leq 1{,}5$ mm) rotary cutting parameters it is logical to suppose that the solution of the mentioned task with the developed methods will be incorrect. Therefore, this work offers researching parameters for rotary deep-depth turning without simplifying assumptions.

2. Methods

Figure 1 shows a scheme for defining the parameters of rotary deep-depth turning and their symbols [1, 3-5]: φ – horizontal cutter of the taper angle; β – vertically dipped cutter angle; h – cutter tool point position in relation to the feed plane going through the axis of the workpiece; B – crown of the cutter; C – crown on circumferential cutting edge of the cutter; A – cutter`s entry point of contacting with the workpiece; M – AB contact of the arc current point of the cutter with the workpiece; Θ – cutter contact angle with the workpiece; Ψ_M – current angle parameter defining the M point position on the contact arc; H – point C position in relation to the plane.

Figure 1 shows two frames: XYZ axes and X´Y´Z´ axes. The frame of axes X,Y,Z is connected to the workpiece and the frame of axes X´Y´Z´ is connected to the cutter [1]. Axis OX coincides with the axis of the workpiece and is directed towards the longitudinal feed. Axis OY is in the base plane. Axis OY is on the basic plane. Point B, the crown of the cutter, is in plane YOZ. Point O, central point, is on the axis of the workpiece.

The central point in the frame of axes X´Y´Z´ is combined with point O', the cutter`s center. A conversion from (one) frame of axes to the other one is done according to the formulas [2]:

$$\begin{aligned} X &= a + a_{11}X' + a_{21}Y' + a_{31}Z'; \\ Y &= b + a_{21}X' + a_{22}Y' + a_{32}Z'; \\ Z &= c + a_{13}X' + a_{23}Y' + a_{33}Z'. \end{aligned} \quad (1)$$

Where a, b, c are the coordinates of the cutter`s center in system XYZ.

The matrix of linear transformation consists in the rotating coordinate system in space at angles φ and β:

$$c = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} \tag{2}$$

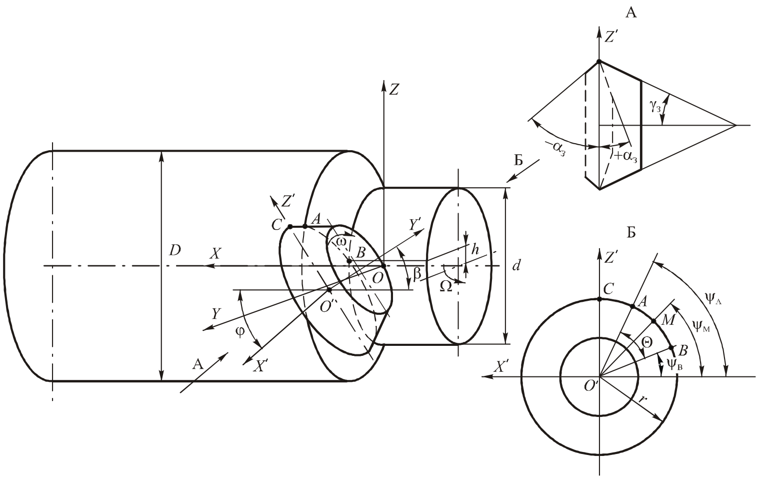

Figure 1. Scheme of rotary deep-depth (t) turning

In our case coefficients a_{ij} are expressed through the setting parameters in the following way:

$$c = \begin{vmatrix} \cos\varphi & -\sin\varphi\cdot\cos\varphi & \sin\varphi\cdot\sin\varphi \\ \sin\varphi & \cos\varphi\cdot\cos\varphi & -\cos\varphi\cdot\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{vmatrix} \tag{3}$$

The coordinates of point O in system XYZ will accordingly be:

$$\begin{aligned} X_B &= 0; \\ Y_d &= \sqrt{\frac{d^2}{4} - h^2}; \\ Z_B &= h. \end{aligned} \tag{4}$$

where d – a diameter of the machined surface. A tangent line vector to the cutting edge at point B has coordinates {-X´$_B$,0, Z´$_B$ }

The equation of the tangent plane to the cylinder of the machined surface at point B:

$$Y_B \cdot Y + Z_B \cdot Z = \frac{d^2}{4}. \tag{5}$$

This is an equation of this plane in system X'Y'Z' taking into account that (4):

$$\begin{aligned} &\sqrt{\frac{d^2}{4} - h^2}\cdot\sin\varphi x' + \left(\sqrt{\frac{d^2}{4} - h^2}\cdot\cos\varphi\cdot\cos\varphi + h\cdot\sin\beta\right)\cdot y' + \\ &+\left(-\sqrt{\frac{d^2}{4} - h^2}\cdot\cos\varphi\cdot\sin\varphi + h\cdot\cos\beta\right)\cdot Z' + \sqrt{\frac{d^2}{4} - h^2}\cdot b + h\cdot c = 0 \end{aligned} \tag{6}$$

The criterion for the parallelism of tangent to the cutting edge and the plane can be written like this:

$$Z'_B\sqrt{\frac{d^2}{4}-h^2}\cdot\sin\varphi+X'_B\left(\sqrt{\frac{d^2}{4}-h^2}\cdot\cos\varphi\cdot\sin\varphi-h\cdot\cos\beta\right)=0 \qquad (7)$$

The equation for the cutting edge of the cutter is:

$$\begin{aligned}&X'_B+Z'^2_B=r^2;\\&Y_B=0\end{aligned} \qquad (8)$$

Having solved equations (7) and (8) simultaneously we obtain the coordinates of cutter`s point B crown in system X´Y´Z´:

$$X'_B=\frac{r\cdot\sin\varphi\sqrt{\frac{d^2}{4}-h^2}}{\sqrt{\frac{d^2}{4}-h^2\cdot\sin^2\varphi+\left[\sqrt{\frac{d^2}{4}-h^2}\cdot\cos\varphi\cdot\sin\varphi-h\cdot\cos\beta\right]^2}};. \qquad (9)$$

$$Y'_B=\frac{r\cdot\left(\sqrt{\frac{d^2}{4}-h^2}\cdot\cos\varphi\cdot\sin\varphi-h\cdot\cos\beta\right)}{\sqrt{\frac{d^2}{4}-h^2\cdot\sin^2\varphi+\left[\sqrt{\frac{d^2}{4}-h^2}\cdot\cos\varphi\cdot\sin\varphi-h\cdot\cos\beta\right]^2}} \qquad (10)$$

$$Z'_B=\frac{r\cdot\left(\sqrt{\frac{d^2}{4}-h^2}\cdot\cos\varphi\cdot\sin\varphi-h\cdot\cos\beta\right)}{\sqrt{\left(\frac{d^2}{4}-h^2\right)\cdot\sin^2\varphi+\left[\sqrt{\frac{d^2}{4}-h^2}\cdot\cos\varphi\cdot\sin\varphi-h\cdot\cos\beta\right]^2}} \qquad (11)$$

An angle parameter, which defines the position of point B on the cutting edge, can be calculated like this:

$$tg\psi_B=\frac{Z'_B}{X'_B}=\frac{\sqrt{\frac{d^2}{4}-h^2}\cdot\cos\varphi\cdot\sin\varphi-h\cdot\cos\beta}{\sin\varphi\cdot\sqrt{\frac{d^2}{4}-h^2}} \qquad (12)$$

Let us simplify the expression (12) and insymbol

$$tg\zeta=\frac{h}{\sqrt{\frac{d^2}{4}-h^2}}.$$

Thus we will finally have:

$$tg\psi_B=ctg\varphi\cdot\sin\beta-tg\zeta\cdot\frac{\cos\beta}{\sin\varphi} \qquad (13)$$

We will find parameters a, b, c from system (1) when substituting values X_B, Y_B, Z_B, X'_B, Y'_B, Z'_B:

$$a = \frac{r \cdot \left[\cos\varphi \cdot \sin\varphi \cdot \sqrt{\frac{d^2}{4} - h^2} - \sin\varphi \cdot \cos\beta (\cos\varphi \cdot \sin\beta - tg\zeta \cdot \cos\beta) \right]}{\sqrt{\sin^2\varphi + (\cos\varphi \cdot \sin\beta - tg\zeta \cdot \cos\beta)^2}} \qquad (14)$$

$$b = \sqrt{\frac{d^2}{4} - h^2} + \frac{r(\sin^2\varphi + \cos^2\varphi \cdot \sin^2\beta - tg\zeta \cdot \cos\beta \cdot \cos\varphi \cdot \sin\beta)}{\sqrt{\sin^2\varphi + (\cos\varphi \cdot \sin\beta - tg\zeta \cdot \cos\beta)^2}} \qquad (15)$$

$$c = h \frac{r \cdot \cos\beta (\cos\varphi \cdot \sin\beta - tg\zeta \cdot \cos\beta)}{\sqrt{\sin^2\varphi + (\cos\varphi \cdot \sin\beta - tg\zeta \cdot \cos\beta)^2}} \qquad (16)$$

Expressions (14), (15), (16) can be written in a simpler and more practically convenient form:

$$a = r \cdot (\cos\varphi \cdot \cos\psi_B - \sin\psi_B \cdot \sin\varphi \cdot \sin\beta) \qquad (17)$$

$$b = r \cdot (\cos\varphi \cdot \cos\psi_B - \sin\psi_B \cdot \sin\varphi \cdot \sin\beta) + \sqrt{\frac{d^2}{4} - h^2} \qquad (18)$$

$$c = h - r \cdot \cos\beta \cdot \sin\psi_B \qquad (19)$$

It is necessary to know the position of the highest cutter`s point C with known parameters φ, β, h, r, d when designing rotary cutters and making experiments. The coordinates of point C in system X´Y´Z´ will be {0, 0, r}. We obtain the following from equation 3 of simultaneous equation (1):

$$H = c + r \cdot \cos\beta \qquad (20)$$

The angle of a cutter`s angle with a workpiece can be defined if we know the values of angles ψ_B and ψ_A. We can calculate angle ψ_A from the conditions that point A of the cutting edge belongs to the cylinder of the surface to be machined:

$$Y^2 + Z^2 = \left(\frac{d}{2} + t \right)^2, \qquad (21)$$

where t – depth of cutting.

Equation (20) will look like this in the system of coordinates X´Y´Z´.

$$(b + a_{12}X' + a_{22}Y' + a_{33}Z')^2 + (c + a_{13}X' + a_{23}Y' + a_{33}Z')^2 = \left(\frac{d}{2} + t \right)^2 \qquad (22)$$

Let us solve equations (8) and (22) simultaneously but in relation to point A:

$$X'_A + Z'_A = r^2;$$
$$Y'_A = 0$$

And when substituting coordinate values $X'_A = -r \cdot \cos\psi_A$, $Y'_A = 0$, $Z'_A = r \cdot \sin\psi$ into equation (19) we will obtain:

$$(b - r \cdot \sin\varphi \cdot \cos\psi_A - r \cdot \cos\varphi \cdot \sin\beta \cdot \sin\psi_A)^2 + (c + r \cdot \cos\beta \cdot \sin\psi_A)^2 = \left(\frac{d}{2} + t \right)^2 \qquad (23)$$

Transcendental equation (23) in relation to parameter ψ_A can be transformed to the algebraic equation of degree 4 with the help of trigonometric rearranging. It is quite time-consuming to solve the equation of degree 4 directly. Here we use the known methods to simplify the solution of this task. However, equation (23) was solved without any simplifying assumptions in relation to ψ_A and with the use of the computer to avoid inaccuracy when defining the contact arc.

The angle of contact Θ is defined the following way:

$$\Theta = \psi_A - \psi_B \qquad (24)$$

3. Results and discussion

Figure 2 shows the dependences of contact angle Θ on the depth of cutting. These dependences are calculated with the simplifying assumptions (curve 1) and without them (curve 2). The dependences imply that the method of approximate solution of equation (22) leads to essential inaccuracies in defining the value of contact angle Θ if we have deep depths of cutting. The bigger is the depth of cutting, the higher is the inaccuracy. So, the inaccuracy can reach 98% if depth of cutting t is 15 mm.

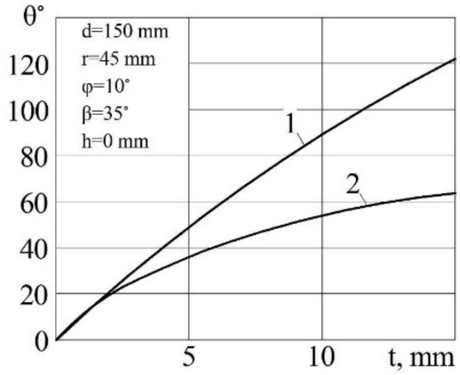

Figure 2. Dependence of depth of cutting t on contact angle θ
(1 – with simplifying assumptions; 2 – without the assumptions)

The inaccuracy in defining the contact angle is higher if setting angle φ increases and setting angle β decreases (Figure 3).

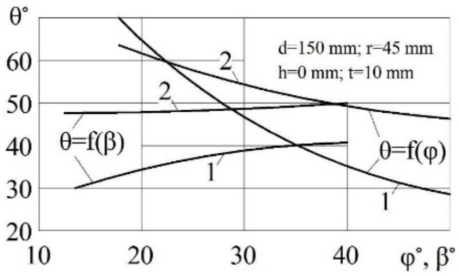

Figure 3. Influence of setting angles β and φ on the inaccuracy when defining contact angle θ
(see symbols in Figure 2)

One of the most essential parameters in the process of rotary turning is the value of ratio between cutter rotary velocities V_{tool} and workpiece to be machined V_w. This ratio is accepted to be called as kinematic coefficient K.

The importance of ratio V_{tool} / V_w is explained by the fact that the ratio predefines power, temperature and wear resistant characteristics of the cutting process as well as the value of sliding velocity of tool`s useful areas in relation to the material to be machined. The kinematic coefficient is defined theoretically without simplifying assumptions:

$$K = \frac{1}{\Theta R_W}\begin{bmatrix} c \cdot \sin\varphi(\cos\psi_A - \cos\psi_B) - r \cdot \cos\beta \cdot \sin\varphi \cdot \Theta + \\ (c \cdot \cos\varphi \cdot \sin\beta + b \cdot \cos\beta)(\sin\psi_A - \sin\psi_B) \end{bmatrix}, \tag{25}$$

where R_w – radius of the workpiece to be machined.

Conclusions

This paper is particularly valuable because it describes the kinematic coefficient that allows one to make theoretical analysis of the rotary turning process and also to design cutters with optimal geometric and setting parameters. The experimental check of the obtained dependences for the analytical research of the main parameters of rotary deep-depth turning proved the reliability of the results in case of changing wide-range machining conditions. Disarrangements between experimental data and the data obtained by means of calculation according to the presented dependences did not exceed 5¸8%.

The main results and conclusions are summarised as follows:

- Using the methods of researching rotary turning with simplifying assumptions leads to sufficient inaccuracies and cannot be applied in researching parameters of rotary deep-depth turning.
- The developed method allows making computerized analytical researches of all the parameters of rotary turning with high reliability of the obtained results.

References

[1] Marghitu D 2005 *Academic Press* 792
[2] Kadry S 2013 *Elsevier* 144
[3] Olgun U and Budak E 2013 *Procedia CIRP* **8** 81-87
[4] Kossakowska J and Jemielniak K 2012 *Procedia CIRP* **1** 425-430
[5] Suzuki N 2014 *Procedia CIRP* **14** 125-129